

## Eliminate the old habit

## Adopt this new habit

## Why?

"Let's 'borrow' from the 10's place."

Compose the lower unit value/decompose the higher unit value.

You might say this: "Here we will need to decompose a higher unit value to get the number into the form we want."

Discontinue using the term "borrow." Use "regroup" and "trade" instead. Also try to include the concept of decomposing and composing the higher unit value.

This language prepares students for situations beyond 10's and 1's, and for fractions and beyond. For example, decompose the higher unit value below to change the form of this mixed fraction to access thirds to solve this problem.

$$\begin{array}{r} 3\frac{1}{3} \\ - 2\frac{2}{3} \end{array}$$

In this form, I don't have access to the thirds I need.

$3\frac{1}{3}$  is rewritten into the form of  $2\frac{3}{3} + \frac{1}{3}$  or  $2\frac{4}{3}$  so that I have access to the thirds that are composed into a whole. The problem then becomes:

$$\begin{array}{r} 2\frac{4}{3} \\ - 2\frac{2}{3} \end{array}$$

In this form, I can readily subtract.

When I decompose the higher unit (ones) into the lower unit (thirds), I am acting on the same mathematical structure as with 10's and 1's. Our language should connect those ideas so that it does not appear to be a novel concept when taught in later grades (Ma, 2010).

Multiplication "makes things get bigger."

Teach the different structures of multiplication.

There are three main structures for multiplication. One is that multiplication is repeated addition (measurement model); another answers the question of how many unique possibilities there are when matching one set with another (fundamental counting principle); the third finds a total amount or area when a column and row or two sides are known (area model).

Try creating a story problem for each different structure type with the expression  $4 \times 3$ .

Multiplication only makes things get bigger in the limited world of positive whole numbers. As with addition and subtraction, focusing on the false idea that an operation does something to something else distracts from conversations about the structures of the given operation.

Discussions about things 'getting bigger' may also distract students from the larger point of an equality. For instance  $2 \times 3 = 6$ . The biggest mathematical thought here is that two expressions are equal to each other, not that something has 'gotten bigger.'

Also, consider  $4 \times \frac{1}{2}$  or  $4 \times -1$ . In both cases, the product has a lesser value than the first factor (Ma, 2010).

Division "makes things get smaller."

Teach the different structures of division.

There are three main structures for division. One structure involves repeated subtraction of groups (measurement model); the second answers the question of "how many for each one?" (partitive model/unit rate model); and the third model for division (area model) involves finding a side when an area and another side are known.

As with multiplication, division has just as much chance of making things smaller as it does of making things larger. In early grades, with only whole numbers to consider, it looks as if division behaves this way, or has that power. But this limited sample makes us think a behavior exists that does not exist. Your divisor, in combination with the act of dividing, determines the relative size of the quotient compared to the size of the dividend, not the operation of division itself. And again, remember that the overall equality between the expressions gets lost when we discuss the false idea that something has 'gotten smaller.'

Consider:

$$6 \div \frac{1}{2} = 12 \quad \text{OR} \quad -6 \div -2 = 3$$

In both cases here, the quotient is a larger number than the dividend and, in both cases, it is important to note that the expressions are equal.

"Doesn't go into."

For example: "7 doesn't go into 3."

Prepare students for future learning

You might say this: "We can divide 7 by 3, but the result won't be a whole number. When you begin working with fractions, you will solve problems like this regularly. Here we want to consider numbers that divide into other numbers without creating fractional parts or leftover pieces."

"Consider if we had seven cookies and needed to split them between three people, what would happen? What if we had nine cookies between three people? In both cases, we can split up the cookies, but which one is easier? Why?"

Again, we need to make sure we maintain our accurate mathematical language even when something will always be true at our grade level. It isn't true that "7 doesn't go into 3." Even young children can understand the idea that in some cases there is a cookie left over that needs to be cut up in order for everyone to have equal shares. They know intuitively that I can have seven cookies and split them between three people. The language that says "you can't do that" separates their intuitive understanding with their academic mathematical understanding. Our job is to connect the intuitive with the academic.